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WAVES ON A VISCOUS-FLUID FILM FLOWING DOWN A VIBRATING VERTICAL PLATE

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A thin film of a viscous fluid flowing down a vertical plane in a gravitational field is considered. The plane executes harmonic oscillations in the direction normal to itself. An equation that describes the evolution of surface disturbances at small fluid flow rates is obtained. Some solutions of this equation are found.

1. Formulation of the Problem. We consider a viscous-fluid film flow down a vertical plane under the action of a gravitational force. The coordinate system used is shown in Fig. 1. The rigid wall along which the film flows executes harmonic oscillations along the y-axis with a constant frequency ω_0 and acceleration amplitude A_0. In the noninertial reference system attached to the wall, the system of Navier–Stokes equations for the film flow has the form

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+g+\nu\Big(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\Big),$$
$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\Big(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\Big)+A_0\cos(\omega_0 t+\varphi),\qquad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{1.1}$$

Here u and v are the velocity components along the x- and y-axis, respectively, ν is the kinematic viscosity, g is the free-fall acceleration, and ρ is the fluid density.

For all fluid flow rates, system (1.1) admits of a solution with a flat free boundary. For this solution, the streamwise velocity and pressure profiles are

$$U_0=\frac{g}{2\nu}(2h_0y-y^2),\qquad P_0=p_a+\rho(y-h_0)A_0\cos(\omega_0 t+\varphi). \tag{1.2}$$

Here h_0 is the thickness of the fluid film and p_a is the atmospheric pressure.

However, already at the smallest flow rates the flow (1.2) can become wavy owing to its instability. Using the characteristic values of the quantities for the case of a wave-free film flow as nondimensionalizing scales, in particular, the thickness h_0 and the free-surface velocity $U_s = gh_0^2/(2\nu)$, we have the following dimensionless form of the equation of motion (the sign of nondimensionalization is omitted here):

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\mathrm{Fr}}\Big(-\frac{\partial p}{\partial x}+1\Big)+\frac{1}{\mathrm{Re}}\Big(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\Big), \tag{1.3}$$
$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\mathrm{Fr}}\Big(-\frac{\partial p}{\partial y}+\frac{A_0}{g}\cos(\omega t+\varphi)\Big)+\frac{1}{\mathrm{Re}}\Big(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\Big),\ \ \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0.$$

The dynamic boundary conditions on the solid ($y = 0$) and free [$y = h_0 + h(x,t)$] boundaries can be written in the following form adopted in [1]:

$$u=v=0,\qquad y=0; \tag{1.4}$$

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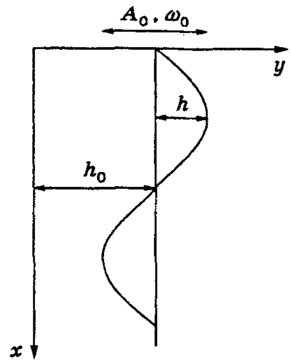

Fig. 1

$$\left(p - \frac{\mathrm{We}}{R}\right)\frac{\partial h}{\partial x} + \frac{\mathrm{Fr}}{\mathrm{Re}}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2\frac{\partial u}{\partial x}\frac{\partial h}{\partial x}\right) = p_a\frac{\partial h}{\partial x},$$
$$p - \frac{\mathrm{We}}{R} + \frac{\mathrm{Fr}}{\mathrm{Re}}\left[\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\frac{\partial h}{\partial x} - 2\frac{\partial v}{\partial y}\right] = p_a, \qquad y = h_0 + h. \tag{1.5}$$

Here $\dfrac{1}{R} = -\dfrac{\partial^2 h}{\partial x^2}\left[1 + \left(\dfrac{\partial h}{\partial x}\right)^2\right]^{-3/2}$, $\omega = \dfrac{\omega_0 h_0}{U_s}$, $\mathrm{Re} = \dfrac{U_s h_0}{\nu}$ is the Reynolds number, $\mathrm{Fr} = \dfrac{U_s^2}{g h_0}$ is the Froude number, $\mathrm{We} = \dfrac{\sigma}{\rho g h_0^2}$ is the Weber number, and σ is the surface-tension coefficient.

On the free boundary, the kinematic condition

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} = v \tag{1.6}$$

is valid.

Using (1.2), it can be easily shown that, for the chosen scales of nondimensionalization, the relation $\mathrm{Re}/\mathrm{Fr} = 2$ is satisfied. Restricting ourselves to the consideration of long-wave disturbances, we seek the solution of system (1.3) under conditions (1.4)–(1.6) in the form of certain series in the small parameter $\varepsilon = h_0/\lambda$ (λ is the characteristic streamwise length of the disturbances). Following [1], we introduce the new variables $x' = \varepsilon x$, $y' = y$, and $\tau_n = \varepsilon^n t$ ($n = 1, 2, \ldots$) and the functions $u = U_0 + \varepsilon u'$, $v = \varepsilon^2 v'$, $p = P_0 + \varepsilon p'$, and $h = \varepsilon h'$.

Neglecting the terms of the order of ε^2 and higher and transferring boundary conditions from the free surface to the undisturbed level, we have the following system of equations for u', v', p', and h' (the primes are omitted):

$$\varepsilon\left(\frac{\partial u}{\partial \tau_1} + U_0\frac{\partial u}{\partial x} + v\frac{dU_0}{dy}\right) = -\frac{\varepsilon}{\mathrm{Fr}}\frac{\partial p}{\partial x} + \frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial y^2}, \quad -\frac{1}{\mathrm{Fr}}\frac{\partial p}{\partial y} + \frac{\varepsilon}{\mathrm{Re}}\frac{\partial^2 v}{\partial y^2} = 0, \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1.7}$$

with the boundary conditions

$$u = v = 0, \qquad y = 0, \tag{1.8}$$

$$\frac{d^2 U_0}{dy^2}h + \frac{\partial u}{\partial y} + \varepsilon\frac{\partial^2 u}{\partial y^2}h = 0, \tag{1.9}$$
$$p + \varepsilon\frac{\partial p}{\partial y}h + \frac{A_0}{g}h\cos(\omega t + \varphi) + \mathrm{We}\,\varepsilon^2\frac{\partial^2 h}{\partial x^2} - 2\varepsilon\frac{\mathrm{Fr}}{\mathrm{Re}}\frac{\partial v}{\partial y} = 0, \qquad y = 1.$$

The kinematic condition (1.6) takes the form

$$\frac{\partial h}{\partial \tau_1} + \varepsilon\frac{\partial h}{\partial \tau_2} + U_0\frac{\partial h}{\partial x} + \varepsilon u\frac{\partial h}{\partial x} = v + \varepsilon\frac{\partial v}{\partial y}h, \qquad y = 1. \tag{1.10}$$

In (1.9), only terms of the highest order in ε are retained, since, normally, the values of We for thin films of many fluids are rather high. Therefore, we assume that the relation $\mathrm{We}\,\varepsilon^2 \sim 1$ is valid.

The solution of system (1.7) can be represented as a series in the parameter ε:

$$(u, v, p, h) = \sum_{k=0}^{\infty} \varepsilon^k (u_k, v_k, p_k, h_{k+1}).$$

Equating the coefficients at the same powers of ε, from the zero-order equations we obtain

$$u_0 = \frac{y}{2}\,h_1, \quad v_0 = -\frac{y^2}{2}\,\frac{\partial h_1}{\partial x}, \quad p_0 = -\frac{A_0}{g}\,h_1 \cos(\omega t + \varphi) - \mathrm{We}\,\varepsilon^2\,\frac{\partial^2 h_1}{\partial x^2}. \tag{1.11}$$

Substituting (1.11) into (1.10), we have the following equation, which describes the behavior of the disturbances in the first-order approximation:

$$\frac{\partial h_1}{\partial \tau_1} + 2\,\frac{\partial h_1}{\partial x} = 0. \tag{1.12}$$

As follows from (1.12), in this first-order approximation all disturbances propagate at a velocity equal to the doubled flow velocity at the flat free boundary.

For the next order in ε, from system (1.7) we have

$$\begin{gathered}\frac{\partial u_0}{\partial \tau_1} + U_0\,\frac{\partial u_0}{\partial x} + v_0\,\frac{dU_0}{dy} = -\frac{1}{\mathrm{Fr}}\,\frac{\partial p_0}{\partial x} + \frac{1}{\mathrm{Re}}\,\frac{\partial^2 u_1}{\partial y^2}, \\ -\frac{1}{\mathrm{Fr}}\,\frac{\partial p_1}{\partial y} + \frac{1}{\mathrm{Re}}\,\frac{\partial^2 v_0}{\partial y^2} = 0, \qquad \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0.\end{gathered} \tag{1.13}$$

For this order, from Eqs. (1.18) and (1.9) we obtain the boundary conditions

$$u_1 = v_1 = 0, \qquad y = 0; \tag{1.14}$$

$$\frac{d^2 U_0}{dy^2} h_2 + \frac{\partial u_1}{\partial y} = 0, \quad p_1 + \frac{\partial p_0}{\partial y} h_1 + \mathrm{We}\,\varepsilon^2 \frac{\partial^2 h_2}{\partial x^2} - 2\frac{\mathrm{Fr}}{\mathrm{Re}}\,\frac{\partial v_0}{\partial y} = 0, \quad y = 1. \tag{1.15}$$

The kinematic condition (1.10) for the given order in ε takes the form

$$\frac{\partial h_1}{\partial \tau_2} + \varepsilon\,\frac{\partial h_2}{\partial \tau_1} + U_0\,\frac{\partial h_2}{\partial x} + \varepsilon u_0\,\frac{\partial h_1}{\partial x} = v_1 + \frac{\partial v_0}{\partial y}\,h_1, \qquad y = 1. \tag{1.16}$$

The solution of system (1.13), which satisfies conditions (1.14) and (1.15), is easy to find. In particular, for the velocity u_1 and for its value v_1 on the boundary, we have

$$u_1 = 2\Big(\frac{\partial h_1}{\partial x}\,\frac{A_0}{g}\cos(\omega t + \varphi) + \mathrm{We}\,\varepsilon^2\,\frac{\partial^3 h_1}{\partial x^3}\Big)(y - y^2/2) - \mathrm{Re}\,\frac{\partial h_1}{\partial \tau_1}\,(y - y^3/3) - \mathrm{Re}\,\frac{\partial h_1}{\partial x}\,(y/3 - y^4/12),$$

$$v_1(y=1) = -\frac{2}{3}\Big(\frac{\partial^2 h_1}{\partial x^2}\,\frac{A_0}{g}\cos(\omega t + \varphi) + \mathrm{We}\,\varepsilon^2\,\frac{\partial^4 h_1}{\partial x^4}\Big) + \frac{5}{12}\,\mathrm{Re}\,\frac{\partial^2 h_1}{\partial \tau_1 \partial x} + \frac{3}{20}\,\mathrm{Re}\,\frac{\partial^2 h_1}{\partial x^2}.$$

Substituting these relations into condition (1.16) with allowance for (1.12), after some simple calculations we obtain

$$\frac{\partial h_1}{\partial \tau_2} + 4h_1\,\frac{\partial h_1}{\partial \zeta} + \Big(\frac{8}{15}\,\mathrm{Re} + \frac{2A_0}{3g}\cos(\omega_1 \tau_2 + \varphi)\Big)\frac{\partial^2 h_1}{\partial \zeta^2} + \frac{2}{3}\,\mathrm{We}\,\varepsilon^2\,\frac{\partial^4 h_1}{\partial \zeta^4} = 0. \tag{1.17}$$

Here $\zeta = x - 2\tau_1$ and $\omega_1 = \omega/\varepsilon^2$. Equation (1.17) is written in the reference system moving with the velocity of linear disturbances that propagate over the film surface in the case of a stationary wall.

Substituting

$$\zeta = b\xi, \quad b = \Big(\frac{5\,\mathrm{We}\,\varepsilon^2}{4\,\mathrm{Re}}\Big)^{1/2}, \quad \omega_1 = \Omega b^4, \quad h_1 = \frac{8b\,\mathrm{Re}\,H}{15}, \quad \tau_2 = \tau b^4, \quad A = \frac{5A_0}{4g\,\mathrm{Re}}$$

into Eq. (1.17), we obtain

$$\frac{\partial H}{\partial \tau} + 4H\frac{\partial H}{\partial \xi} + [1 + A\cos(\Omega\tau + \varphi)]\frac{\partial^2 H}{\partial \xi^2} + \frac{\partial^4 H}{\partial \xi^4} = 0. \tag{1.18}$$

For the case of a stationary wall ($A = 0$), Eq. (1.18) reduces to

$$\frac{\partial H}{\partial \tau} + 4H\frac{\partial H}{\partial \xi} + \frac{\partial^2 H}{\partial \xi^2} + \frac{\partial^4 H}{\partial \xi^4} = 0, \tag{1.19}$$

which, by convention but not perfectly right, is called the Kuramoto–Sivashinskii equation. The latter differ from (1.19) by its nonlinear term, but the solutions of the two equations can be easily expressed through each other, and, therefore, we will also call Eq. (1.19) the Kuramoto–Sivashinskii equation. As applied to the study of descending films, Eq. (1.19) was derived in [1]. Up to the present time, it has been studied rather carefully, and many solutions have been constructed.

Similarly to the Kuramoto–Sivashinskii equation, the periodic or localized solutions of (1.18) retain their "momentum"

$$\frac{\partial}{\partial \tau}\int_0^\lambda H\,d\xi = 0.$$

Here λ is the wavelength, and, for the case of a localized disturbance, the integration limits from $-\infty$ to $+\infty$ are assumed.

Multiplying (1.18) by H and integrating the resulting equation, after some rearrangements, we obtain the following relation for the evolution of "energy," which is valid for any periodic (solitary) solution of (1.18):

$$\frac{1}{2}\frac{\partial}{\partial \tau}\int_0^\lambda H^2\,d\xi = [1 + A\cos(\Omega\tau + \varphi)]\int_0^\lambda \left(\frac{\partial H}{\partial \xi}\right)^2 d\xi - \int_0^\lambda \left(\frac{\partial^2 H}{\partial \xi^2}\right)^2 d\xi. \tag{1.20}$$

2. Results. Equation (1.18) with the nonlinear term omitted describes the stability of the initial flow (1.2) against infinitesimal disturbances. These solutions are easy to find. Representing them as spatially-uniform waves $H = F(\tau)\exp(i\alpha\xi) + \text{c.c.}$, we obtain

$$H = F_0\exp[i\alpha\xi + \alpha^2((1-\alpha^2)\tau + A/\Omega\sin(\Omega\tau + \varphi))] + \text{c.c.} \tag{2.1}$$

Here α is the wavenumber, F_0 is an arbitrary complex constant, and c.c. means a complex-conjugate expression. It follows from (2.1) that, in the case of both vibrating and stationary walls, the disturbances with wavenumbers smaller than unity are unstable. If the wavenumber is greater than unity, the disturbances decay with time. For the wavenumber $\alpha_n = 1$, solution (2.1) remains finite as the time τ increases. In the adopted reference system, it is a standing wave whose amplitude executes oscillations at the frequency Ω of wall vibrations.

To analyze the nonlinear periodic solutions of Eq. (1.18), we solved this equation numerically. The solution was represented in the form of a spatial Fourier series with time-dependent harmonics:

$$H(\xi, \tau) = \sum_{n=-\infty}^{\infty} H_n(\tau)\exp(in\alpha\xi). \tag{2.2}$$

By virtue of the reality of H, the relation $H_{-n} = H_n^*$ holds for the harmonics H_n. Here the asterisk means a complex-conjugate quantity.

Substituting (2.2) into (1.18), we obtain an infinite system of nonlinear ordinary differential equations for H_n. Terminating series (2.2), i.e., assuming that all harmonics H_n from a certain number N equal zero, we obtain the following finite analog of the system:

$$\frac{dH_n}{d\tau} = -4i\alpha n\sum_{m=N-n}^{N} H_m H_{n-m} + \alpha^2 n^2\{[1 + A\cos(\Omega\tau + \varphi)] - \alpha^2 n^2\}H_n, \quad n = 0, 1, \ldots, N. \tag{2.3}$$

From (2.3) it follows that the equation in the harmonic H_0 is trivial and H_0 equals an arbitrary constant. In what follows, we will construct solutions for which this constant equals zero. In other words, we will consider the solutions for which the quantity H averaged over the wavelength is equal to zero. Taking into account that Eq. (1.18) is invariant relative to the transform $H \to H + \text{const}$, $\xi \to \xi - 4\,\text{const}\,\tau$ and $\tau \to \tau$, we conclude that all other solutions can be obtained from the solutions listed below by simple recalculation.

Thus, separating the real and imaginary parts in (2.3) and taking into account that $H_{-n} = H_n^*$, we have a system of $2N$ ordinary differential equations in $2N$ unknown functions.

System (2.3) was solved numerically by a fifth-order Runge–Kutta method with an automatic choice of the integration step and a controlled accuracy. In terminating series (2.3), the number of harmonics was taken to satisfy the relation $\max|H_n|/\sup|H_n| < 10^{-4}$. For the results listed below, as the calculations show, N = 5–6 is sufficient. The main results are obtained with $N = 10$. To verify some solutions obtained, we also solved the system with $N = 25$. A direct comparison showed that, for the first three harmonics (for the considered wavenumbers α, the contribution due to these harmonics was determining), only the fourth to fifth decimal digit was different.

At the present time, there are many papers devoted to the Kuramoto–Sivashinskii equation. The pattern of its solution is extremely rich. For instance, it was shown [1] that for $\alpha = 1$ a family of periodic solutions of Eq. (1.19) appears, which deviates from the trivial solution and can be extended to the wavenumber $\alpha_* = 0.4979$. This family is traditionally called the first family. In the same work [1], for this family a range of wavenumbers $0.77 \leqslant \alpha \leqslant 0.84$ was established throughout which the solutions are stable against all small perturbations. Against small disturbances of the same period, the solutions of the first family with the wavenumbers $0.554 \leqslant \alpha \leqslant 1$ are stable. It was shown [2–4] that there is a countable set of periodic families of the solutions. Two families were found [5], which for $\alpha \to 0$ transform into solitonlike solutions in the form of a solitary peak and a solitary valley. Multipeak solitonlike solutions of Eq. (1.19) were constructed in [6], and it was shown [3, 4] that such solutions are the limiting ones for the families branching off the families obtained in [5]. In the same study, it was shown that a succession of bifurcations occurs, which gives rise to more intricate families.

The available information on periodic steady-state solutions of the Kuramoto–Sivashinskii equations simplifies the study of the regimes given by Eq. (1.18). Taking into account the complexity of the wave structure of the Kuramoto–Sivashinskii equation, in the present work we restrict ourselves to the construction of only those solutions of Eq. (1.18) whose wavenumbers fall into a close vicinity of the neutral wavenumber $\alpha = 1$. In this case, the behavior of the solutions of the Kuramoto–Sivashinskii equation is most simple. If $\alpha > 1$, all of them, irrespective of the initial waveform assumed, decay in time. If, however, $\alpha < 1$ and no subharmonics in the solution of system (2.3) were permitted (i.e., the wavenumber of the first harmonics was α, and not α/r, where r is an integer number), then all initial wave profiles tend to the corresponding solutions of the first family with time.

As the calculations showed, even in that wavenumber region where the structure of the solution of the Kuramoto–Sivashinskii equation is quite simple, the effect of wall vibrations makes the solution pattern much more diversified. This pattern is largely influenced by both the amplitude and frequency of the vibrations. The effect of the initial phase for the wavenumber region considered is substantially less pronounced. In all examples listed below, it was put equal to $\varphi = -\pi/2$, i.e., the coefficient at the second derivative in Eq. (1.18) had the form $1 + A\sin(\Omega\tau)$.

Curve 1 in Fig. 2 shows the dependence of the absolute value of the amplitude H_1 of the first harmonic on time for the wavenumber $\alpha = 0.95$, $A = 1$, and $\Omega = 1$ (in what follows, the amplitude of a harmonic is understood as its modulus). The initial value of the real part of H_1 is 0.5. Here, as well as in Figs. 3–6, the initial values of all other harmonics and the imaginary part of the first harmonic H_1 are assumed to equal zero. For comparison, curve 2 shows the corresponding solution of the Kuramoto–Sivashinskii equation. Up to the moment $\tau = 20$, it approaches a constant, which coincides with the value obtained, for example, in [3–5] for the steady-state wave regime. It is seen that the solution of Eq. (1.18) rapidly reaches a certain steady level and executes regular oscillations around the steady-state solution of the Kuramoto–Sivashinskii equation. Figure 3 shows the profiles of this solution for three different times within a time interval equal

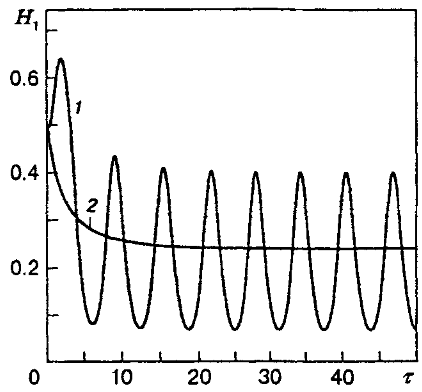

Fig. 2

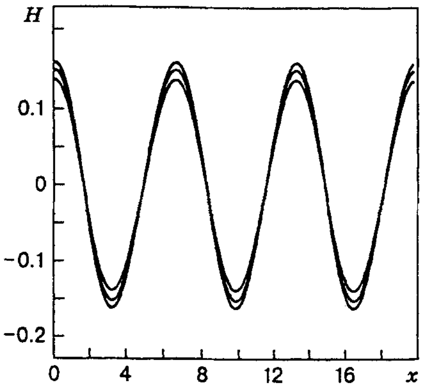

Fig. 3

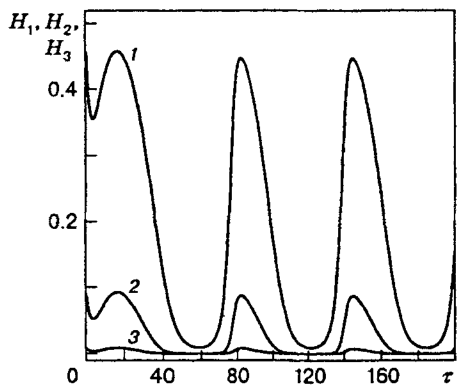

Fig. 4

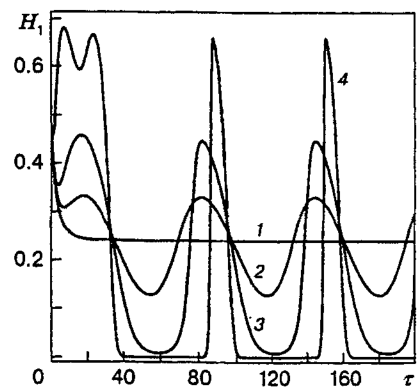

Fig. 5

to the period of wall vibration for $\alpha = 0.95$, $A = 1$, and $\Omega = 1$. Three spatial periods are shown. For each of these moments, the wave is seen to be a perfect sinusoid. As shown in Fig. 2, for this calculation variant the wave amplitude is time-modulated by a nearly perfect sine curve. The latter is due to the fact that, for the solutions with the wavenumbers around the neutral number $\alpha_n = 1$, the amplitudes of higher harmonics rapidly decrease. The orders of these amplitudes can be inferred, for example, from Fig. 4. Here curves 1, 2, and 3 show the time dependence of the amplitudes of the first (H_1), second (H_2), and third (H_3) harmonics, respectively. The wave number is $\alpha = 0.95$, and the amplitude and the frequency of wall vibrations are $A = 0.3$ and $\Omega = 0.1$, respectively.

For a fixed value of frequency ω, the oscillations of the solution become greater and more different from the sinusoid as the amplitude A increases. Figure 5 shows the time dependence of the amplitude of the first harmonic H_1. For all curves, $\alpha = 0.95$. Curve 1 corresponds to $A = 0$ (the solution of the Kuramoto–Sivashinskii equation), curve 2 to $A = 0.1$ and $\Omega = 0.1$, and curve 3 to $A = 0.3$ and $\Omega = 0.1$.

For a given amplitude A, the solution transforms in a similar manner with increasing frequency Ω.

Provided that, at a given amplitude A, the wavenumber α is close to the neutral wavenumber but smaller than it ($\alpha < 1$ is the linear instability region), solutions with an interesting behavior are observed for sufficiently small frequencies of wall vibrations Ω (for curve 4 in Fig. 5, $A = 2$ and $\Omega = 0.1$). It is seen that there are rather long time intervals in which the solution practically equals zero; then it rapidly increases, with the amplitude of the fundamental harmonic reaching a maximum (other harmonics also increase rapidly, but in proper proportions). Afterwards, the solution rapidly decreases and again becomes equal to zero for a long period of time. Except for the first segment (interval τ = 0–40 for curve 4), where the behavior of the solution depends on the initial data, this process is repeated over and over. As a result, we have an interesting auto-oscillatory regime whose characteristics (the "peaceful" periods and the amplitude peaks) are independent of the initial data and depend only on the frequency Ω and amplitude A of wall vibrations.

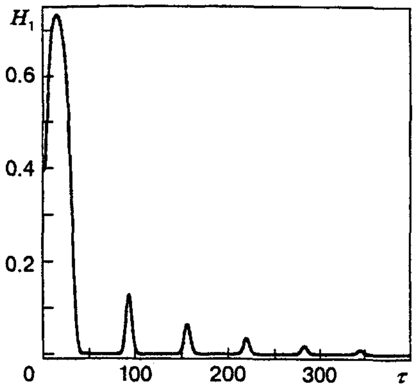

Fig. 6

This behavior of the solution can be explained, however not quite rigorously, as follows. If the frequency Ω is sufficiently small, the phase of the coefficient at the second derivative in Eq. (1.18) can be considered practically constant, i.e., being "frozen" during a fairly long period of time. For this time period, the local linear instability region falls in the interval of wavenumbers

$$0 < \alpha < [1 + A\cos(\Omega\tau + \varphi)]^{1/2}. \tag{2.4}$$

Thus, the upper boundary of the indicated local instability region "breathes" in time. If the wavenumber α lies near α_n, but $\alpha < \alpha_n$, and the amplitude A is sufficiently large, then at those moments when α falls into region (2.4) and the amplitudes of other harmonics are very small, a linear exponential growth of the wave is observed at first (at this stage, the value of the cosine keeps growing), and at the nonlinear stage even a more rapid growth of the wave occurs. Afterwards, when the upper limit in (2.4) starts decreasing, the solution decreases as rapidly as it previously grew. When the wavenumber α falls in the stability region, the stage of linear exponential decrease is observed. Subsequently, the process is repeated over and over again. The above considerations are confirmed by calculation results on modeling the evolution of periodic disturbances with the wavenumbers within the stability region. This calculation is exemplified by Fig. 6, which shows the time evolution of the first-harmonic amplitude for the solution with the wavenumber $\alpha = 1.005$ ($A = 1$ and $\Omega = 0.1$). It is seen from Fig. 6 that, although the solution, as a whole, lies within the stability region, for a certain time interval it falls into the local instability region. This, as in the first case, results in the outbursts of the amplitudes. Nevertheless, the peak values of these outbursts decrease with time, and, as a result, the solution decays. The reason for this difference in the two regimes close to α_n but lying on its opposite sides is that in the first case ($\alpha < \alpha_n$) the solution remains in the local instability region for a longer period of time than in the local stability region, and in the second case ($\alpha > \alpha_n$) quite the opposite situation is observed. A numerical analysis of Eq. (1.20) for the "energy" balance shows that, when $\alpha > \alpha_n$, the losses of "energy" are not compensated by its pumping.

Conclusions. Equation (1.18) obtained in this work allows one to study the behavior of disturbances in a film flow on a vibrating vertical plane for small flow rates of the fluid. The wave regimes of the flow are largely influenced by both the amplitude and frequency of wall vibrations. As the calculations show, even when the wavenumbers of the disturbances are close to the neutral wavenumber α_n, interesting wave regimes can occur on the film surface, which are not observed in the case of a film flowing down a stationary plane.

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